

On the theoretical possibility of the electromagnetic scalar potential wave spreading with an arbitrary velocity in vacuum

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Abstract

In this work we revisit the process of constructing wave equations for the scalar and vector potentials of an electromagnetic field, and show that a wave equation with an arbitrary velocity (including a velocity higher than the velocity of light in vacuum) for the scalar potential exists in the framework of classical electrodynamics. Some consequences of this fact are considered.

Our letter is devoted to the discussion of a possibility of the existence of sub- and superluminal electromagnetic waves in vacuum.

A considerable number of experimental and theoretical works about superluminal spreading of electromagnetic waves, particles and other objects have recently been published, as mentioned by E. Recami in [1,2], Walker in [3], Kotel'nikov in [4,5] and in the book edited by Chubykalo *et al* [6] (see also references in the mentioned works). J. Marangos wrote in his brilliant note “Faster than a speeding photon” [7]: “*The textbooks say nothing can travel faster than light, not even light itself. New experiments show that is no longer true, raising questions about the maximum speed at which we can send information.*” (see also the bibliography in this work). Really, a series of recent experiments, performed at Cologne[8], Berkeley[9], Florence[10] and Vienna[11], and quite recent experiments by W. Tittel *et al* [12] revealed that evanescent waves (in undersized waveguides, e.g.) seem to spread with a superluminal group velocity. For example, in up-to-date experiments by Mugnai *et al* (see their work [13]) superluminal behavior in the propagation of microwaves (centimeter wavelength) over much longer distances (tens of centimeters) at a speed 7% faster than c was reported.

In the majority of cases, these works almost directly declare that generally accepted electrodynamics must be sufficiently reconsidered.¹

In this paper we would like to address the problem of electromagnetic waves spreading with an *arbitrary* velocity in vacuum ($v \leq c$ and $v \geq c$) conditioned by *a choice of gauges*. In other words, we attempt to explain here these superluminal electromagnetic phenomena using well-known arbitrariness of a choice of gauges permitted by classical

¹See incidentally “Essay on non-Maxwellian theories of electromagnetism” by V. Dvoeglazov [14].

electrodynamics.

Is there a way of showing a possibility of the existence of the superluminal wave processes in vacuum mentioned in the beginning of the letter, without leaving the framework of classical electrodynamics? Here we show that, yes, there is.

It is known, if we are given potentials \mathbf{A} and φ , then these uniquely determine the fields \mathbf{E} and \mathbf{H} :

$$\mathbf{E} = -\nabla\varphi - \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t} \quad \text{and} \quad \mathbf{H} = \nabla \times \mathbf{A}. \quad (1)$$

However, it is well known that the same field can correspond to different potentials. Electric and magnetic fields determined from equations (1) actually do not change upon replacement of \mathbf{A} and φ by \mathbf{A}' and φ' , defined by

$$\mathbf{A}' = \mathbf{A} + \nabla f \quad \text{and} \quad \varphi' = \varphi - \frac{1}{c}\frac{\partial f}{\partial t}. \quad (2)$$

Only those quantities which are invariant with respect to the transformation (2) have physical meaning, so all equations must be invariant under this transformation.

This non-uniqueness of the potentials gives us the possibility of choosing them so that they fulfill one auxiliary condition (gauge) chosen by us. It means that we can set one, since we may choose the function f in (2) arbitrarily.

Let us recall how a necessity of choosing a gauge condition arises. One substitutes fields \mathbf{E} and \mathbf{H} , expressing by potentials \mathbf{A} and φ , into Maxwell equations

$$\nabla \cdot \mathbf{E} = 4\pi\varrho, \quad (3)$$

$$\nabla \cdot \mathbf{H} = 0, \quad (4)$$

$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}, \quad (5)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}, \quad (6)$$

to obtain equations which are easier-to-use than the original equations with respect to fields \mathbf{E} and \mathbf{H} . After direct substituting (1) into Maxwell equations we have:

$$\Delta\varphi + \frac{1}{c}\frac{\partial}{\partial t}\text{div } \mathbf{A} = -4\pi\varrho \quad (7)$$

and

$$\Delta \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} - \text{grad} \left(\text{div} \mathbf{A} + \frac{1}{c} \frac{\partial \varphi}{\partial t} \right) = -\frac{4\pi}{c} \mathbf{j}. \quad (8)$$

Thus because of non-uniqueness of the potentials, we can always subject them to an auxiliary condition. For this reason, we try to choose this condition so as the system of equations (7) and (8) (or at least one of them) would be transformed into some “easy-to-solve” equations. Let us consider this problem from a purely formal, mathematical point of view. Note if a certain connection between $\text{div} \mathbf{A}$ and $\frac{\partial \varphi}{\partial t}$ exists then because of dimensions condition this connection must generally look like

$$\text{div} \mathbf{A} + \alpha \frac{\partial \varphi}{\partial t} = 0, \quad (9)$$

where α is an *arbitrary* constant parameter with dimensions $\left[\frac{1}{\text{cm/sec}} \right]$. We will show that the condition (9) can satisfy (2) by a corresponding choice of function f in (2). For this we substitute values \mathbf{A}' and φ' into (9):

$$\nabla \mathbf{A} + \Delta f + \alpha \frac{\partial \varphi}{\partial t} - \frac{\alpha}{c} \frac{\partial^2 f}{\partial t^2} = 0. \quad (10)$$

Let us make now a formal conjecture that a perturbation of potential φ spreads with a certain *arbitrary* constant velocity v (for a given process) which is not necessarily equal to c . We choose the arbitrary constant α as $\alpha = \frac{c}{v^2}$ then the condition (9) becomes

$$\text{div} \mathbf{A} + \frac{c}{v^2} \frac{\partial \varphi}{\partial t} = 0, \quad (11)$$

.

Consequently from (10) we obtain the equation for f :

$$\Delta f - \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2} = F(\mathbf{r}, t), \quad (12)$$

where $F(\mathbf{r}, t) = -\nabla \mathbf{A} - \frac{c}{v^2} \frac{\partial \varphi}{\partial t}$ is a given function \mathbf{r} and t . Substituting the function f from a solution of Eq. (12) into formulas (2) we find values of potentials \mathbf{A}' and φ' satisfying *gauge* (11)².

²note that if we choose the arbitrary constant α as $\alpha = \frac{1}{c}$ instead of gauge (11), we obtain the well-known Lorentz gauge

$$\text{div} \mathbf{A} + \frac{1}{c} \frac{\partial \varphi}{\partial t} = 0.$$

The gauge (11) unlike the Lorentz gauge, has no relativistically invariant character. But gauge (11) is obviously allowable in classical electrodynamics *on parity with other* well-known gauges: the Coulomb gauge

$$\operatorname{div} \mathbf{A} = 0 \quad \text{and} \quad \Delta\varphi = -4\pi\rho \quad (13)$$

and the so-called radiation gauge

$$\operatorname{div} \mathbf{A} = 0 \quad \text{and} \quad \varphi = 0, \quad (14)$$

which in turn also have no relativistically invariant character [15].

Let us apply the gauge (11) to Eqs. (7) and (8). As a result we obtain two equations:

$$\Delta\varphi - \frac{1}{v^2} \frac{\partial^2 \varphi}{\partial t^2} = -4\pi\rho \quad (15)$$

and

$$\Delta\mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = \operatorname{grad} \left(\frac{v^2 - c^2}{cv^2} \frac{\partial \varphi}{\partial t} \right) - \frac{4\pi}{c} \mathbf{j}. \quad (16)$$

In current- and charge-free regions these equations become

$$\Delta\varphi - \frac{1}{v^2} \frac{\partial^2 \varphi}{\partial t^2} = 0 \quad (17)$$

and

$$\Delta\mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = \left(\frac{v^2 - c^2}{cv^2} \right) \operatorname{grad} \frac{\partial \varphi}{\partial t}. \quad (18)$$

We can see that the equations obtained differ strongly from the well-known wave equations for charge-free space in the Lorentz gauge

$$\Delta\varphi_L - \frac{1}{c^2} \frac{\partial^2 \varphi_L}{\partial t^2} = 0 \quad (19)$$

and

$$\Delta\mathbf{A}_L - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}_L}{\partial t^2} = 0 \quad (20)$$

(index L denotes that these potentials obey Lorentz gauge). Actually, a solution of Eq.(17) is a wave of perturbation of the scalar potential spreading in vacuum with the arbitrary

phase velocity³ v which can be both lower than c and higher than c (unlike the solution of Eq.(19)!) Then there is Eq.(18) which is a wave equation with a source (unlike Eq.(20)) despite the fact that this equation is written for charge(current-)-free space. The function of source in Eq.(18) is a gradient of changing of the solution of Eq.(17) with changing of time. Note that the gauge (11) is more general gauge than Coulomb and Lorentz gauges. Actually, if we choose the arbitrary constant velocity v in (11) as c we obtain the Lorentz gauge and corresponding wave equations for potentials (19),(20), on the other hand if v in (11) tends to infinity we immediately obtain the Coulomb gauge and corresponding equations for potentials:

$$\Delta\varphi_C = -4\pi\rho \quad (21)$$

and

$$\Delta\mathbf{A}_C - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}_C}{\partial t^2} = \text{grad} \left(\frac{1}{c} \frac{\partial \varphi_C}{\partial t} \right) - \frac{4\pi}{c} \mathbf{j}. \quad (22)$$

(index C denotes that these potentials obey Coulomb gauge).

Obviously, equations (15)-(18) and specially Eqs. (15) and (17) can lead the way to found a theory of superluminal electromagnetic interactions and signal transfer in vacuum, which experimental evidences were mentioned in the Introduction. And the scalar potential φ from Eqs. (17) has to play the leading role in constructing this theory.

Against this, one can say that there is the well-known, generally accepted opinion in classical electrodynamics that electromagnetic potentials are just auxiliary quantities which have no real physical meaning (unlike real mensurable magnitudes \mathbf{E} and \mathbf{H}). We can reply to this retort as follows:

On the one hand, from *generally accepted* classical electrodynamics we know that the Poynting vector is *proportional* to the density of the electromagnetic field momentum. But on the other hand, paradoxes connected with the Poynting vector (and, correspondingly, with an energy and momentum distribution) exist and they are well-known. For example, in [16] it is noted: if a point charge Q is vibrating in some mechanical way along the X -axis with respect to a certain point x_Q , then the value of electromagnetic energy density

³recall that in vacuum, the phase velocity and the group velocity of electromagnetic perturbation spreading coincide.

w (which is a point function like \mathbf{E}) on the same axis will be also oscillating. Immediately the question arises: *how does the test charge q at the point of observation, lying at some fixed distance from the point x_Q along the continuation of the X -axis, “know” about the charge Q vibration?* In other words, we have a rather strange situation: the Poynting vector $\mathbf{S} = \frac{c}{4\pi}[\mathbf{E} \times \mathbf{H}]$ is zero along this axis⁴ (because \mathbf{H} is zero along this line) but the energy and the momentum, obviously “pass” from point to point along this axis. This means that *we cannot be sure any more* that exclusively using the vector fields \mathbf{E} and \mathbf{H} allows us to characterize the process of distributing the electromagnetic energy and momentum in vacuum within the framework of classical electrodynamics. It would be very interesting to carry out thorough research in this problem. At this stage, we can only say that it is unlikely that one can solve this problem without taking into account a *physical reality* of electromagnetic potentials waves. On the other hand the equation (17) and (18) are conjunct, i.e. the radiator (source) in the wave equation (18) for the vector potential is the function of the superluminal (for example) solution of (17)! What does it mean theoretically? It means that *the source of the vector potential’s waves spreads with an superluminal velocity*. But the vector potential in turn produces the variable magnetic field (it follows from $\nabla \times \mathbf{A} = \mathbf{H}$). The variable magnetic field in turn produces the variable electric field. It means that a perturbation of the scalar potential started up in some distant point reaches the given point and produces in it *measurable* fields \mathbf{E} and \mathbf{H} .

It is an accepted truth that, for the purpose of long-range radio communications, one employs mainly transverse electromagnetic waves propagating at the speed of light. These waves are described mathematically by relatively simple equations whose consequences can be easily verified by experiment. On the other hand, as it was noted in the recent work [18], there exists an extensive area of electromagnetic phenomena characterized by a complicated and mainly approximate mathematical formulation whose consequences cannot be easily tested experimentally⁵. This is precisely the area where one can expect to find

⁴Note that recently many works devoted to the Poynting vector concept was published. See, e.g., [17] and corresponding references there.

⁵Optical phenomena are not excluded, since we know nothing about the details of quantum transitions responsible for the emission of photons by atoms.

unknown effects and unusual properties of the electromagnetic field. Such an expectation is supported by the fact that even classical electrodynamics does not actually prohibit either the superluminal velocity or the longitudinal electromagnetic waves if we take into account *the physical reality* of potentials and existence of Eqs. (15)-(18). It is clear, therefore, that in general electromagnetic fields may propagate in a manner drastically different from that with which we are familiar on the basis of radio broadcasting.

For objectiveness we have to mention that many works are published where authors claim that the absolute majority of experiments discovered the superluminal propagation are incorrect (see, for example, the book [6] the Section “Contra”). But we just would like to note that it cannot be emphasized enough that in the problem of the experimental confirmation or confutation of the superluminal propagation the last word has not yet been said on this matter. But now we have the *formal proof*, based only on Maxwell electrodynamics, that electromagnetic waves connected with a perturbation of scalar potential φ (Eqs. (15), (17)) *can* travel either faster than the speed of light in vacuum c or slower than c but not exclusively with c exactly! Here let us observe (following E.Recami [19]) that the particular role of the speed of light in the Special Relativity is due to its *invariance*, and not to the fact that it is (or it is not) the *maximal* one.

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Appendix ⁶

Recently, two works [20, 21] on the subject considered in this paper appeared, where it is stated that despite of non-retarded origin of the scalar potential in any gauge (*except the Lorentz one*), the \mathbf{E} field calculated in *any gauge* is retarded. To the authors’ point of view, it is not so.

⁶This Chapter is added after sending the manuscript

One can easily see that in both cited works ⁷, the key point of proof of equivalence of the gauges is to show that the current density \mathbf{j} can be presented as a sum of two currents, longitudinal and transversal:

$$\mathbf{j}_{\parallel} + \mathbf{j}_{\perp} = \mathbf{j} \quad (23)$$

by the way, the longitudinal current, which is the source in the equation for the vector potential, is yielded by the *action-at-distant* scalar potential so it is described by *highly nonlocal function* (Eq. (3.9) of [21] and Eq. (2.8) of [20]).

$$\mathbf{j}_{\parallel} = -\frac{1}{4\pi} \nabla \int \frac{\nabla' \cdot \mathbf{j}(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r}' \quad (24)$$

Correspondingly, the transversal current is described by highly nonlocal function too:

$$\mathbf{j}_{\perp} = \frac{1}{4\pi} \nabla \times \nabla \times \int \frac{\mathbf{j}(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r}' \quad (25)$$

Eqs. (24) and (25) correspond to Eqs. (6.49) and (6.50) of [22], where the proof that the sum of *rhs* of these equations is equal to the *rhs* of Eq. (23) is given. This proof is based on application of the vector identity

$$\nabla \times \nabla \times \mathbf{j} = \nabla(\nabla \cdot \mathbf{j}) - \nabla^2 \mathbf{j} \quad (26)$$

and equation $\nabla^2(1/|\mathbf{r} - \mathbf{r}'|) = -4\pi\delta(\mathbf{r} - \mathbf{r}')$. However, it should be noted that while proving validity of Eq. (23), (or Eq. (6.48) of [22]), two formal rules are broken here, *i.e.*

- In Eq. (25) the differential operator $\nabla \times \nabla \times$ must act on the vector too but $1/|\mathbf{r} - \mathbf{r}'|$ is a scalar,
- the differential operator $\nabla \times \nabla \times$ acts on external variable but the vector equality (26) must be applied to internal variables.

So, the proof given in [22] is incorrect and, therefore, calculations of works [20, 21] based on this proof are incorrect too.

⁷We focus on the Coulomb gauge as it is done in [20, 21], but basically it can be shown for any gauge

Physical explanation of incorrectness of this proof is simple: original current density is the *local* quantity. After Eq. (23), it is proposed to form the local quantity from two nonlocal quantities. Let us consider a point of space where the current is zero. We should create from this *zero* two vector fields; by the way one of them is *rotational* (Eq. 25)) and second is *irrotational* (Eq. (24)), *i.e.* two fields different in their origin. So the rotational field is equal to 'minus' irrotational field and, because for the vector the sign "minus" correspond to opposite direction of the vector, *rotational field* = - *irrotational field*, that is nonsense.

The reader can see that we do not use Eq. (23) in our calculations and just in this point, there is *essential difference* between this work and all previous works on the gauges in the classical electrodynamics where correctness of Eq. (23) is undeniably accepted.

REFERENCES

- [1] E. Recami, *Physica A* **252**, 586 (1998).
- [2] A.P.L.Barbero, H.E.Hernández-Figueroa, & E.Recami, *Phys. Rev. E* **62** 8628 (2000);
A.P.L. Barbero, H.E. Hernández-Figueroa, and E. Recami, “On the propagation speed of evanescent modes” in: *Lorentz Group, CPT and Neutrios*, ed. by A. Chubykalo, V. Doeglazov, D. Ernst, V. Kadyshevsky, and Y.S. Kim (World Scientific, Singapore 2000), p. 305.
- [3] W.D. Walker, “Superluminal near-field dipole electromagnetic fields” in: *ibid*, p. 327.
- [4] G.A. Kotel’nikov, *Soviet Physics Journal* **24**(10), 938 (1981).
- [5] G.A. Kotel’nikov, “The sign inversion of the speed of light is the new transformation of discrete symmetry in electrodynamics” in: *Photon and Poincaré Group*, ed. by V. Dvoeglazov (Nova Science, New York 1999), p. 128.
- [6] *Instantaneous Action at a Distance in Modern Physics: ‘Pro’ and ‘Contra’*, ed. by A. Chubykalo, V. Pope and R. Smirnov-Rueda (Nova Science, New York 1999).
- [7] J. Marangos, *Nature* **406**, 243 (2000).
- [8] W. Heitmann and G. Nimtz, *Phys. Lett. A* **196**, 154 (1994).
- [9] A.M. Steinberg, P.G. Kwiat, and R.I. Chiao, *Phys. Rev. Lett.* **71**, 708 (1993).
- [10] A. Ranfagni, P. Fabeni, G. Pazzi, and D. Mignai, *Phys. Rev. E* **48**, 1453 (1993).
- [11] Ch. Spielmann, R. Szipocs, A. Stingl, and F. Rrausz, *Phys. Rev. Lett.* **73**, 2308 (1994).
- [12] W. Tittel *et al*, *Phys. Rev. Lett.* **81**, 3563 (1998).
- [13] D. Mugnai, A. Ranfagni, and R. Ruggeri, *Phys. Rev. Lett.* **84**, 4830 (2000).
- [14] V. V. Dvoeglazov, *Hadronic J. Suppl.* **12**, 241 (1997).
- [15] Landau L.D and Lifshitz E.M., *Teoria Polia* (Nauka, Moscow, 1973) [English translation: *The Classical Theory of Field* (Pergamon, Oxford, 1975)]
- [16] A. E. Chubykalo and R. Smirnov-Rueda, *Phys. Rev. E* **53** 5373 (1996); see also the *Errata*, *Phys. Rev. E* **55** 3793 (1997); and the discussion: Ivezić T. and Škovrlj L., *Phys. Rev. E* **57**(3), 3680 (1998); A. E. Chubykalo and R. Smirnov-Rueda, *Phys. Rev. E* **57**(3), 3683 (1998).
- [17] V.S. Ilyin and I.S. Nefedov, “Instantaneous momentum-energy of the classical free electromagnetic field” in: *Photon: Old Problems in Light of New Ideas*, ed by V.

- Dvoeglazov (Nova Science Publishers, Inc. 2000), p. 57.
- [18] G. F. Ignatiev and V. A. Leus, “On a superluminal transmission at the phase velocities” in: *Instantaneous Action at a Distance in Modern Physics: ‘Pro’ and ‘Contra’*, ed. by A. Chubykalo *et al* (Nova Science, New York 1999), p. 203.
 - [19] E. Recami, “Relativity and Beyond” in: *Astrofisica e Cosmologia Gravitazione Quanti e Relativita (Negli sviluppi del pensiero scientifico di Albert Einstein. ”Centenario di Einstein” 1879-1979)* (Giunti Barbera, Firenze, 1979) (in Italian).
 - [20] F. Rohrlich, Causality, the Coulomb field, and Newton’s law of gravitation. *Am. J. Phys.* **70** (4), 411, 2002.
 - [21] J.D. Jackson, From Lorenz to Coulomb and other explicit gauge transformations, arXiv.org: physics/0204034
 - [22] J.D. Jackson J D, *Classical Electrodynamics*, 2nd edn. (New York: Wiley, 1975),